Macroeconomic and monetary policies from the "eductive" viewpoint.

R. Guesnerie.
Collège de France, Paris School of Economics.

November 5th, 2007, Preliminary.

1 Introduction

The "quality" of coordination of expectations is a key issue for monetary policy. The question involves different, but interrelated channels, involving the "credibility" of the Central Bank intervention and the ability of decentralized agents to coordinate on a dynamical equilibrium.

Unsurprisingly, the understanding of the learning process of the agents is a key ingredient of the analysis of the quality of expectational coordination. Many studies focus attention on "evolutive", real time learning rules (adaptive learning rules, etc…). The "eductive" viewpoint, as illustrated in many references of this bibliography and in my 2005 MIT Press book, partly abstracts from the real time dimension of learning, with the aim of exhibiting more directly the systems' characteristics that are coordination-friendly.

The objective of the paper is to confront the methods and philosophy of analysis of expectational coordination, that refer to what I just called the "eductive" viewpoint and the actual method and philosophy that underly most present studies of learning in the context of macroeconomic and monetary policy. The paper aims primarily at giving a synthetical flavour of the "eductive" viewpoint as well as presenting a brief review of existing results in the context of dynamical systems Existing applications of the "eductive" method to macroeconomics bear on general equilibrium (2) or dynamical systems (3) [1]but not directly on monetary policy issues. The exploration of the differences between the traditional viewpoint and this other viewpoint in standard monetary policy models is extremely tentative, although it seems to me potentially promising This text will hopefully generate new reflection in the directions stressed

The paper will proceed as follows:

[1]See in particular, Guesnerie R. (2001) ""Short run expectational coordination: Fixed versus flexible wages." Quarterly Journal of Economics, p. 1115,1147, Evans G., R. Guesnerie (2005) "Coordination on saddle path solutions: the eductive viewpoint, 2 - Linear multivariate models, Journal of Economic Theory, 2005, p.202-229.

- It will recall the logic of the "eductive" viewpoint and stress differences as well as complementarities with the "evolutive" viewpoint.
- It will contrast the viewpoints for the theory of abstract dynamical systems, emphasizing the problem of heterogeneity of expectations.
- It will select a sample of models for which it will start comparing the standard learning viewpoint and the so-called "eductive" approach.

2 Expectational stability : the "eductive viewpoint".

The notion of "eductively stable" equilibrium or "strongly rational equilibrium" relies on considerations that have a game-theoretical underpinning, and refer to "rationnalizability", "dominance solvability", "Common Knowledge" ideas. This provides a "high tech" justification of the expectational stability criteria that are proposed. I first put emphasis on this "high tech" approach for proposing global concepts of expectational stability (2-A). I turn then to the local transposition of the global ideas and stress that the criteria have now, besides the previous "high tech" justification a low tech, intuitive interpretation (2-B). I finally comment on the connections between the "eductive" viewpoint and the standard "evolutive" learning viewpoint (2-C).

2.1 Global "eductive" stability.

We are in a world populated of rational economic agents, (in all the following, I shall assume that these agents are infinitesimal and associated with a continuum[2]), rationality of the agents is Common Knowledge and so are the interactions between them (the model is Common Knowledge, from now on CK). The state of the system is denoted E and belongs to some subset $\mathcal{E}$ of some vector space. An equilibrium of the system is a state E* such that if everybody believes that it prevails, it does prevail. (Note that this implies in particular that the assertion it is CK that E=E* is not absurd).

Note that E can be a number, (the value of an equilibrium price in Guesnerie (1992) or quantity Guesnerie (2001) or a growth rate, in Evans-Guesnerie(2003)), a vector (of equilibrium prices, or quantities, in Guesnerie (2005), Chapter 6), a function, (the equilibrium demand function in many finance models see Desgranges (2000), Heinemann (2004), Desgranges-Heinemann (2005), Ben Porath(2006) or an infinite trajectory of states, (in Evans-Guesnerie (2005), a probability distribution.in Desgranges-Gauthier(2003)

Let us make a presentation which is both abstract (although not fully explicit) and synthetical

[2] The mathematical difficulties and specificities of the continuum, and the connection of rationalizability in this setting and in standard game theoretical setting is analysed in Jara (2007).

Guesnerie-Jara (2007) also àbtainrather genereal results on global "eductive" stability.

We say that E* is "eductively" stable of "strongly rational" iif Assertion A implies assertion B.

Assertion A : It is CK that E $\in \mathcal{E}$ (and implicitely that Bayesian rationality and the model are CK)

Assertion B : it is CK that E=E*.

The mental process that leads from Assertion A to Assertion B is the following.

1- As every body knows that E $\in \mathcal{E}$, everybody knows that everybody limits its responses to actions that best responses to some probability distributions over $\mathcal{E}$. It follows that everybody knows that the state of the system will be in $\mathcal{E}(1) \subset \mathcal{E}$

2- If $\mathcal{E}(1)$ is a proper subset of $\mathcal{E}$, the mental process goes on as in step 1, but with $\mathcal{E}(1)$ instead of $\mathcal{E}$.

3- etc...

We then have a (weakly) decreasing sequence $\mathcal{E}(n) \subset \mathcal{E}(n-1) \subset .... \subset \mathcal{E}(1) \subset \mathcal{E}$. When the sequence converges to E*, the equilibrium is strongly, rational or "eductively" stable. When it is not the case, the limit set is the set of rationalizable equilibria of the model. (See Guesnerie-Jara-Moroni (2007)).

Global "eductive" stability is clearly very demanding, although it can be shown to hold under plausible economic conditions in a variety of models, either partial equilibrium (Guesnerie (1992)), general equilibrium (Guesnerie (2001)), finance and transmission of information through prices (Desgranges-Geoffard-Guesnerie (2002)), or in general settings involving strategic complementarities or substitutabilities (Guesnerie-Jara-Moroni(2007).

2.2 Local "eductive" Stability

Local "eductive stability may be defined through the same 'high tech" or hyperrationality view (2B-1). However, the local criterion has also a very intuitive and low tech and boundedly rational interpretation (2B-2).

2.2.1 Local "eductive" stability as a CK statement.

We say that E* is locally "eductively" stable or locally "strongly rational" iif one can find some non trivial neighbourhood of E*, $V(\mathcal{E})$ such that Assertion A implies assertion B.

Assertion A : It is CK that E $\in V(\mathcal{E})$

Assertion B : it is CK that E=E*.

Hypothetically, the state of the system is assumed to be in some non-trivial neighbourhood of E*, and this hypothetically CK assumption implies the CK of E*.

In other words, the deletion of non-best responses, starts under the assumption that the state of the system is close to the equilibrium state. In that sense, the viewpoint refers to the same hyper-rationality view as referred to before. However, the statement can be read in a simpler way.

2.2.2 Local "eductive" stability as a common sense requirement.

It seems intuitively plausible to define local expectational stabilty as follows : there exists a non trivial neighbourhood of the equilibrium such that if everybody believes that the state of the system is in this neighbouhood, whatever the specific form taken by eveybody's belief, it is the case that the state is in the stressed neighbouhood. Intuitively absence of such a neighbourhood signals some tendancy to instability : there can be facts falsifying any conjecture on the set of possible states, unless this set reduces to the equilibrium itself.

Naturally, it is easy to check, and left to the reader, that the failure of getting local "expectational stability" is a failure of the above local intuitive requirement.

2.3 "Eductive" versus "evolutive" learning stability.

There is an informal argument, due to Milgrom-Roberts (1990), according to which, in a system that repeats itself, non best responses to existing observations will be deleted after a while, initiating a "real time" version of the notional time deletion of non-best responses underlying "eductive" reasoning. Let us focus here on the connections between local "eductive stability" and the local convergence of "evolutive" learning rules. What the "eductive stability" involves is that once, for whatever reasons, the (possibly stochastic) beliefs of the agents will be trapped in $V(\mathcal{E})$, they will remain in $V(\mathcal{E})$, as soon as the updating process is let us say, Bayesian. Although it is not quite enough to be sure any "evolutive" learning rule will converge, it is the case that in many settings, one can show that local "eductive" stability involves that every "reasonable" evolutive real time learning rule converges asymptotically (see Guesnerie (2002) , Gauthier-Guesnerie (2005),). Furthermore, it should be clear that the failure ot find a set $V(\mathcal{E})$ for which the the equilibrium is locally strongly rational, signals a tendancy for reasonable states of beliefs, close to the equilibrium, and then probably compatible with some reasonable evolutive updating, to be triggered away in some cases, a fact that threatens the convergence of the corresponding learning rule.

Hence, our very abstract and hyper-rational criterion, provides a short cut for understanding the difficulties of expectational coordination, without entering into the business of specifying the real time, bounded rationality considerations that may matter. Naturally, the "eductive" criterion is in general more demanding than most fully specified "evolutive" learning rules one can think of (see previous references).

In cases of models with "extrinsic uncertainty", the equilibrium is a probability distribution, a state of the system in the sense of the word taken here is a probability distribution. An observation is not an observation on the state in our sense, but an information on the state in the standard sense of the word. "Evolutive and "eductive learning may differ significantly..

3 "Eductive" versus "evolutive" learning in infinite horizon models.

Models used for monetary policy generally adopt an infinite horizon approach. This section reviews existing results on "eductive" stability in infinite horizon models. It is based on Desgranges-Gauthier (2002) Gauthier (), Evans-Guesnerie (), Gauthier-Guesnerie (). The review will allow to confront the game-theoretically oriented viewpoint stressed here with the standard macroeconomic approach to the problem as reported in Evans-Honkappohja (2001).

3.1 Standard expectational analysis in one-dimensional one step-forward memory one models.

3.1.1 The model

Consider a one-dimensional model in which the state of the system to-day is determined form its value yesterday and its expected value to-morrow, according to the linear (for the sake of simplicity) equation :

$$\gamma E\left[x(t+1) \mid I_t\right] + x(t) + \delta x(t-1) = 0,$$

where x is a one-dimensional variable γ and δ are real parameters $(\gamma, \delta \neq 0)$.[3].

A perfect foresight trajectory is a sequence $(x(t), t \geq -1)$ such that

$$\gamma x(t+1) + x(t) + \delta x(t-1) = 0$$

in any period $t \geq 0$, given the initial condition $x(-1)$.

Assume that the equation $g_1 = -\gamma g_1^2 - \delta$ has only two real solutions λ_1 and λ_2 (which arises if and only if $1-\delta\gamma \geq 0$) with different moduli (with $|\lambda_1| < |\lambda_2|$ by definition). Therefore, given an initial condition $x(-1)$, there are two perfect foresight solutions : $x(t) = \lambda x(t-1)$,i.e $x(t) = \lambda_1 x(t-1)$. and $x(t) = \lambda_2 x(t-1)$.

The steady state sequence $(x(t) = 0, t \geq -1)$ is a perfect foresight equilibrium if and only the initial state $x(-1)$ equals 0. The steady state is a sink if $|\lambda_2| < 1$, a saddle if $|\lambda_1| < 1 < |\lambda_2|$, or a source if $|\lambda_1| > 1$. We focus attention here on the so-called saddle-path case : the solution $x(t) = \lambda_1 x(t-1)$, generally called the saddle path has been stressed for a long time by economists as the focal solution, on the basis of arguments that refer to expectational plausibility. We review, first, the standard expectational criteria that are used and confirm that the saddle-path solution fit them.

3.1.2 The standard expectational criteria.

Determinacy. The first criterion is determinacy. Determinacy means that the equilibrium under consideration is locally isolated. In our infinite horizon setting, determinacy has to be viewed as a property of trajectories : a trajectory

[3]Such dynamics obtain from linearized versions of overlapping generations models with production, at least for particular technologies (Reichlin (1986)), etc....

$(x(t), t \geq -1)$ is determinate if there is no other trajectory $(x'(t), t \geq -1)$ that is close to it. This calls for a reflection about the notion of proximity of trajectories, i.e to the choice of a topology. Yet the choice of the suitable topology is open. The most natural candidate is the C0 topology, according to which two different trajectories $(x(t), t \geq -1)$ and $(x'(t), t \geq -1)$ are said to be close whenever $|x(t) - x'(t)| < \varepsilon$, for any $\varepsilon > 0$ arbitrarily small, and any date $t \geq -1$. In fact, with such a concept of determinacy, the saddle-path solution, along which $x(t) = \lambda_1 x(t-1)$ when $|\lambda_1| < 1 < |\lambda_2|$, is the only non-explosive solution to be locally determinate in the C0 topology.

Growth rates determinacy. In the present context of models with memory, a saddle solution is characterized by a constant *growth rate* of the state variable This suggests that determinacy should be applied in terms of growth rates, in which case closedness of two trajectories $(x(t), t \geq -1)$ and $(x'(t), t \geq -1)$ would require that the ratio $x(t)/x(t-1)$ be close to $x'(t)/x'(t-1)$ in each period $t \geq 0$. This is an ingredient of a kind of C1 topology, as advocated by Evans and Guesnerie (2003a). In this topology, two trajectories $(x(t), t \geq -1)$ and $(x'(t), t \geq -1)$ are said to be close whenever both the levels $x(t)$ and $x'(t)$ are close, and the ratios $x(t)/x(t-1)$ and $x'(t)/x'(t-1)$ are close, in any period.

As stressed for example by Gauthier (2002), the examination of proximity in terms of growth rates leads consideration of the dynamics with perfect foresight in terms of growth rates.

Define

$$g(t) = x(t)/x(t-1),$$

For any $x(t-1)$ and any $t \geq 0$, then the perfect foresight dynamics implies :

$$x(t) = -\left[\gamma g\left(t+1\right) g\left(t\right) + \delta\right] x(t-1)$$

Or

$$g\left(t\right) = -\left[\gamma g\left(t+1\right) g\left(t\right) + \delta\right]$$

Associated with the initial perfect foresight dynamics, is then a *perfect foresight dynamics of growth rates*. The growth factor $g\left(t\right)$ is determined at date t by the correct forecast of the next growth factor $g\left(t+1\right)$. This new dynamics is non-linear, and it has a one-step forward looking structure, without predetermined variable.

We have then reassessed the problem in terms of one-dimensional one step forward looking models which are more familiar

Sunspots on growth rates Maintaining the focus on growth rates, let us now define a concept of sunspot equilibrium, in the neighborhood of a constant growth rate solution. Suppose that agents a priori believe that the growth factor is to be perfectly correlated with sunspots (sunspots are generated by a Markov process)

Namely, if the sunspot event is s at date t, they a priori believe that $g(t) = g(s)$, that is

$$x(t) = g(s)x(t-1).$$

Thus, their common forecast is

$$E[x(t+1) \mid I_t] = \pi(s,1)g(1)x(t) + \pi(s,2)g(2)x(t),$$

where $\pi(s,1)$ and $\pi(s,2)$ are the sunspot transition probabilities. As shown by Desgranges and Gauthier (2003), this consistency condition is written $g(s) = -[\gamma[\pi(s,1)g(1)+\pi(s,2)g(2)]g(s)+\delta]$.

When $g(1) \neq g(2)$,the formula defines a sunspot equilibrium on the growth rate, as soon as the stochastic dynamics of growth rates is extended as $g(t) = -\gamma E[g(t+1) \mid I_t]g(t) - \delta$. [4]

Evolutive learning on growth rates. It makes sense to learn growth rates from past observations. Agents then update their forecast of the next period growth rates from the observation of past or present actual rates.

Reasonable learning rules in the sense of Gauthier-Guesnerie (2005) consist of adaptive learning rules that are able to "detect cycles of order two".

Iterative Expectational Stability. (IE Stability) We shall refer here to IE-stability criterion (see Evans (1985), de Canio, (1978). Lucas (1979)), and apply it to conjectures on growth rates

Let agents a priori believe that the law of motion of the economy is given by

$$x(t) = g(\tau)x(t-1),$$

where $g(\tau)$ denotes the conjectured growth rate at step τ in some mental reasoning process. Then, they expect the next state varaible to be $g(\tau)x(t)$, so that the actual value is $x(t) = -\delta x(t-1)/(\gamma g(\tau)+1)$. Assume that all the agents understand that the actual growth factor is $-\delta/(\gamma g(\tau)+1)$ when their initial guess is $g(\tau)$, they should revise their guess as

$$g(\tau+1) = -\frac{\delta}{\gamma g(\tau)+1}$$

This is the IE-stability criterion. By definition, IE-stability obtains whenever the sequence $(g(\tau), \tau \geq 0)$ converges toward one of its fixed point, a fact that is interpreted as reflecting the success of some mental process of learning. Since this dynamics is the time mirror of the perfect foresight dynamics of growth rate, a fixed point λ_1 or λ_2 is locally IE-stable if and only if it is locally unstable in the previous growth rates dynamics, that is locally determinate.

3.1.3 Standard criteria versus "eductive stability".

This is, within a simple model, a somewhat careful reminder of the four possible and more or less standard viewpoints on "expectational stability". We want to compare their viewpoints with the so called "eductive viewpoint" emphasized here. The comparison is made easier when one notes that it turns out that here these a priori different approaches of the problem lead to the same result.

[4] Clearly this equivalence relies on special assumptions about linearity and certainty equivalence.

An equivalence theorem on standard "expectational criteria" Proposition . Equivalence principle in one-step forward, memory one, one-dimensional linear systems.

Consider a one-step forward looking model (with one lagged predetermined variable, where $\gamma, \delta \neq 0$. Assume that we are in the saddle-path case. Then the following four statements are equivalent:

1. A constant growth rate solution is locally determinate in the perfect foresight growth rate dynamics and equivalently here in determinate in the C1 topology of trajectories.

2. A constant growth rate solution is locally immune to (stationary) sunspots on growth rates.

3. For any a priori given "reasonable" learning rules bearing on growth rates,constant growth rate solution is locally asymptotically stable.

4. A constant growth rate solutiona is locally IE stable.

In particular, a saddle-path solution meets all these requirements.This is shown in Gauthier-Guesnerie (2005),using previous findings The fact that "reasonable" learning processes converge relies on a definition of reasonableness integrating the suggestions of Grandmont-Laroque (1991) and results of Guesnerie-Woodford (1991).

4 Multidimensional one-step forward looking linear models with memory one

4.1 The framework

We now consider a multidimensional linear one-step forward looking economy with one predetermined variable, formalized as : $\mathbf{G}E\left(\mathbf{x}(t+1) \mid I_t\right) + \mathbf{x}(t) + \mathbf{D}\mathbf{x}(t-1) = \mathbf{o}$,

where $\mathbf{x}$ is a $n \times 1$ dimensional vector, $\mathbf{G}$ and $\mathbf{D}$ are two $n \times n$ matrices, and $\mathbf{o}$ is the $n \times 1$ zero vector.

A perfect foresight equilibrium is a sequence $(\mathbf{x}(t), t \geq 0)$ (a trajectory) associated with the initial condition $\mathbf{x}(-1)$, and such that : $\mathbf{G}\mathbf{x}(t+1) + \mathbf{x}(t) + \mathbf{D}\mathbf{x}(t-1) = \mathbf{o}$.

The dynamics with perfect foresight is governed by the $2n$ eigenvalues λ_i ($i = 1, ..., 2n$) of the following matrix (the matrix associated with the dynamics $(\mathbf{x}(t+1), \mathbf{x}(t))$ as a function of $(\mathbf{x}(t), \mathbf{x}(t-1))$

$$\mathbf{A} = \begin{pmatrix} -\mathbf{G}^{-1} & -\mathbf{G}^{-1}\mathbf{D} \\ \mathbf{I}_n & \mathbf{0} \end{pmatrix},$$

where $\mathbf{0}$ is the n-dimensional zero matrix.

Let by definition $|\lambda_i| < |\lambda_j|$ whenever $i < j$ ($i, j = 1, ..., 2n$). From now, we focus attention on the generalized saddle-point case, where $|\lambda_n| < 1 < |\lambda_{n+1}|$.

In what follows, we consider the perfect foresight dynamics restricted to a n-dimensional eigensubspace, and especially in one spanned by the eigenvectors associated with the n roots of lowest modulus.

Let $\mathbf{u}_i$ denote the eigenvector associated with λ_i $(i = 1, ..., 2n)$. Assuming that all the eigenvalues are distinct, the n eigenvectors form a basis of the subspace associated with $\lambda_1, ..., \lambda_n$. Let:

$$\mathbf{u}_i = \begin{pmatrix} \tilde{\mathbf{v}}_i \\ \mathbf{v}_i \end{pmatrix}$$

where $\mathbf{v}_i$ and $\tilde{\mathbf{v}}_i$ are of dimension n. We check that if $\mathbf{u}_i$ is an eigenvector, then $\tilde{\mathbf{v}}_i = \lambda_i \mathbf{v}_i$.

Hence, if we pick up some $\mathbf{x}(0)$, then if the n-dimensional subspace generated by $(\mathbf{u}_1, ..., \mathbf{u}_n)$is in general position, we can find a single $\mathbf{x}(1) = \sum a_i \lambda_i$ in the subspace and generate a sequence $(\mathbf{x}(t), t \geq 0), (x(2) = \sum a_i \lambda_i \mathbf{u}_i$ following the just defined dynamics. This generates a solution, which is converging in the saddle path case.

The methodology proposed for constructing constant growth rates solution in the previous Section can be replicated to obtain what is called minimum order solutions. Assume that

$$\mathbf{x}(t) = \mathbf{B}\mathbf{x}(t-1) \tag{1}$$

in every period t, and for any n-dimensional vector $\mathbf{x}(t-1)$ ($\mathbf{B}$ is an $n.n$ matrix). Also, $\mathbf{x}(t+1) = \mathbf{B}\mathbf{x}(t)$. Thus, it must be the case that $\mathbf{B} = -(\mathbf{G}\mathbf{B} + \mathbf{I}_n)^{-1}\mathbf{D}$, or equivalently $(\mathbf{G}\mathbf{B} + \mathbf{I}_n)\mathbf{B} + \mathbf{D} = \mathbf{0}$. A matrix $\bar{\mathbf{B}}$ satisfying this equation is what Gauthier (200) calls a stationary *extended growth rate.* [5]

4.1.1 The expectational plausibility of Extended Growth Rates solutions according to standard criteria.

We will concentrate on three of the above criteria : determinacy, immunity to sunspots, and IE-stability.

Determinacy. Determinacy is viewed through a dynamics of *perfect foresight extended growth rates* that extends the dynamics of growth rates previously introduced. Consider for every t, $\mathbf{B}(t)$ a n-dimensional matrix whose ijth entry is equal to $\beta_{ij}(t)$ and $\mathbf{x}(t) = \mathbf{B}(t)\mathbf{x}(t-1)$.

$B(t)$ is a time variable, non-stationnary extended growth rate.

As $\mathbf{x}(t+1) = \mathbf{B}(t+1)\mathbf{x}(t)$,the dynamics with perfect foresight of the endogenous state variable $\mathbf{x}(t)$ induces a dynamics with perfect foresight of extended growth rates $\mathbf{B}(t)$ that is obtained by considering

$$\mathbf{G}\mathbf{B}(t+1)\mathbf{x}(t) + \mathbf{x}(t) + \mathbf{D}\mathbf{x}(t-1) = \mathbf{o}$$

[5] It is shown in Evans and Guesnerie (2003b) that $\bar{\mathbf{B}} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}$, where $\mathbf{\Lambda}$ is a $n \times n$ diagonal matrix whose iith entry is λ_i $(i = 1, \ldots, n)$ and $\mathbf{V}$ is the associated matrix of eigenvectors. In what follows, we focus attention on the saddle-point case, where $|\lambda_n| < 1 < |\lambda_{n+1}|$.

$$\Leftrightarrow \mathbf{x}(t) = -(\mathbf{GB}(t+1) + \mathbf{I}_n)^{-1}\mathbf{D}\mathbf{x}(t-1)$$

,provided that $\mathbf{GB}(t+1) + \mathbf{I}_n$ is a n-dimensional regular matrix. Then, the perfect foresight dynamics is defined by a sequence of matrices $(\mathbf{B}(t), t \geq 0)$ such that :

$$\mathbf{B}(t) = -(\mathbf{GB}(t+1) + \mathbf{I}_n)^{-1}\mathbf{D} \Leftrightarrow (\mathbf{GB}(t+1) + \mathbf{I}_n)\mathbf{B}(t) + \mathbf{D} = \mathbf{0}$$

.This defines the extended growth rates perfect foresight dynamics. Its fixed point are the stationary matrices $\bar{\mathbf{B}}$ such that $\mathbf{B}(t) = \bar{\mathbf{B}}$ in whatever t. Determinacy of the matrix $\bar{\mathbf{B}}$, is standardly defined as the fact that $\bar{\mathbf{B}}$ is locally isolated, i.e that there does not exist a sequence $\mathbf{B}(t)$ of perfect foresight extended growth rates converging to $\bar{\mathbf{B}}$.

Sunspot equilibrium. A sunspot equilibrium on extended growth rates, is defined in the same spirit as for the one-dimensional system. Then, the whole matrix $\mathbf{B}(t)$ that links $\mathbf{x}(t)$ to $\mathbf{x}(t-1)$ has to be exactly correlated with sunspots.

If sunspot event is s $(s = 1, 2)$ at date t, so that

$$E\left(\mathbf{x}(t+1) \mid s\right) = \left[\pi(s,1)\mathbf{B}(1) + \pi(s,2)\mathbf{B}(2)\right]\mathbf{B}(s)\mathbf{x}(t-1).$$
$$\mathbf{x}(t) = -\left[\mathbf{G}\left[\pi(s,1)\mathbf{B}(1) + \pi(s,2)\mathbf{B}(2)\right]\mathbf{B}(s) + \mathbf{D}\right]\mathbf{x}(t-1).$$

In a sunspot equilibrium, the a priori belief that $\mathbf{B}(t) = \mathbf{B}(s)$ is selfulfilling whatever $x(t-1)$, so that :

$$\mathbf{B}(s) = -\left[\mathbf{G}\left[\pi(s,1)\mathbf{B}(1) + \pi(s,2)\mathbf{B}(2)\right]\mathbf{B}(s) + \mathbf{D}\right].$$

Iterative Expectational Stability. What about here, the IE-stability criterion ?

At virtual time τ of the learning process, let assume that agents believe that, whatever t:

$$\mathbf{x}(t) = \mathbf{B}(\tau)\mathbf{x}(t-1),$$

where $\mathbf{B}(\tau)$ is the τth estimate of the n-dimensional matrix $\mathbf{B}$. Their forecasts are accordingly:

$$E\left(\mathbf{x}_{t+1} \mid I_t\right) = \mathbf{B}(\tau)\mathbf{x}_t.$$

The actual dynamics is obtained by reintroducing forecasts into the temporary equilibrium map (**??**):

$$\mathbf{GB}(\tau)\mathbf{x}_t + \mathbf{x}_t + \mathbf{D}\mathbf{x}_{t-1} = \mathbf{o} \Leftrightarrow \mathbf{x}_\tau = -(\mathbf{GB}(\tau) + \mathbf{I}_n)^{-1}\mathbf{D}\mathbf{x}_{\tau-1}.$$

As a result, the dynamics with learning is written:

$$\mathbf{B}(\tau+1) = -(\mathbf{GB}(\tau) + \mathbf{I}_n)^{-1}\mathbf{D}. \tag{2}$$

Comparing this set of equations with the previous one, a stationary EGR $\bar{\mathbf{B}}$ is locally IE-stable if and only if the above dynamics is converging when $\mathbf{B}(0)$ is close enough to $\bar{\mathbf{B}}$.

4.1.2 The dynamic equivalence principle

We can state the following proposition :
Proposition 4. Equivalence principle in one-step forward, memory one, multi-dimensional linear systems.

Consider a stationary EGR

The following three statements are equivalent:

1. The EGR *solution is determinate in the perfect foresight extende growth rates dynamics.*

2. *The* EGR *solution is immune to sunspots, that is, there are no neighbour local sunspot equilibria on extended growth rates with finite support, as defined above.*

3. *The* EGR *solution is locally IE-stable.*

In particular, the saddle-path like solution (that exists when the n smallest eigenvalues of A have modulus less than 1, the $(n+1)th$ has modulus greater than 1) meets all these conditions.

Again, this is proved in Gauthier-Guesnerie (2005)

The flavour of this statement is very close to that of the statement obtained in the one dimensional case.

Note however, that the connection between "evolutive" learning and "eductive" learning is less crystal clear. Adaptive learning processes bearing on the multi-dimensional object extended growth rates is less easy to assess that in the one-dimensional case of previous section.

4.2 "Eductive Stability"

4.2.1 The underlying strategic framework.

The discussion of the basic viewpoint of eductive learning requires that some game theoretical flesh be given to the dynamical model under scrutiny, i.e that embed the dynamic model in a dynamic game as in Evans and Guesnerie (2003b).

We repeat, for the sake of completeness, the presentation of the construct of Evans and Guesnerie (2003).

An OLG context is assumed. At each period t, there exists a continuum of agents. A part of the agents "react to expectations"., another part uses strategies that are not reactive to expectations (in the evoked OLG context, these are the agents, who are at the last period of their lives), The former are denoted ω_t and belong to a convex segment of R, endowed with Lebesgue measure $d\omega_t$.

It is assumed that an agent of period t is different from any other agent of period $t^{'}, t^{'} \neq t$. More precisely, agent ω_t has a (possibly indirect) utility function that depends upon

1) his own strategy $s(\omega_t)$,

2) sufficient statistics of the strategies played by others i.e. on $y_t = F(\Pi_{\omega_t}\{s(\omega_t)\}, *)$, where F in turn depends first, upon the strategies of all agents who at time t react to expectations, and second, upon $(*)$, which is here supposed to be sufficient

statistics of the strategies played by those who do not react to expectations, and that includes but is not necessarily identified with – see below – y_{t-1},

3) finally upon the sufficient statistics for time $t+1$, as perceived at time t: i.e. on $y_{t+1}(\omega_t)$, which *may be random* and, now directly, upon the sufficient statistics y_{t-1}.

Here, strategies played at time t can be made conditional on the equilibrium value of the of the t sufficient statistics y_t. If we denote $(\bullet)$ both (the product of) the probability distribution of the random random subjective forecasts held by ω_t of y_{t+1}, $\tilde{y}_{t+1}(\omega_t)$ and y_{t-1} Let then $G(\omega_t, y_t, \bullet)$ be the best response function of agent ω_t.

Noting that the sufficient statistics for the strategies of agents who do not react to expectations is $(*) = (y_{t-1}, y_t)$, we obtain the time t equilibrium equations :

$$y_t = F\left[\Pi_{\omega_t}\left\{G(\omega_t, y_t, y_{t-1}, \tilde{y}_{t+1}(\omega_t))\right\}, y_{t-1}, y_t\right]$$

.Note that when all agents have the same point expectations denoted y^e_{t+1}, the equilibrium equations determine a kind of temporary equilibrium mapping

$$Q(y_{t-1}, y_t, y^e_{t+1}) = y_t - F\left[\Pi_{\omega_t}\left\{G(\omega_t, y_t, y_{t-1}, y^e_{t+1})\right\}, y_{t-1}, y_t\right].$$

Also assuming that all $\tilde{y}_{t+1}$ have a very small common support "around" some given y^e_{t+1}, as well as the existence of adequate derivatives, decision theory suggests that G, to the first order, depends on the expectation of the random variable $\tilde{y}_{t+1}(\omega_t)$ that is denoted $y^e_{t+1}(\omega_t)$ (and is close to y^e_{t+1}), we are able to linearize around any initially given situation, denoted (0), as follows

$$y_t = U(0)y_t + V(0)y_{t-1} + \int W(0, \omega_t) y^e_{t+1}(\omega_t) d\omega_t,$$

where $y_t, y_{t-1}, y^e_{t+1}(\omega_t)$ now denote small deviations from the initial values of y_t, y_{t-1}, y^e_{t+1}, and $U(0), V(0), W(0, \omega_t)$ are $n \times n$ square matrices.

If such a linearization is considered in a neighbourhood of the steady state, y_t, y_{t-1}, etc., will denote deviations from the steady state and $U(0), V(0), W(0, \omega_t)$ are simply $U, V, W(\omega_t)$.

Adding an, invertibility assumption, we arrive at reduced form :

The temporary equilibrium reduced form, associated with homogeneous expectations,

$$y^e_{t+1}(\omega_t) = y^e_{t+1}, y_t = By^e_{t+1} + Dy_{t-1},$$

And the reduced form associated with stocahstic beliefs

$$y_t = Dy_{t-1} + B\int Z(\omega_t) y^e_{t+1}(\omega_t) d\omega_t$$

,where

$$\int Z(\omega_t) d\omega_t = I.$$

and $t = 1, 2, 3, \ldots$, (initial conditions y_0 being given)

In the present context, this reduced form allows to analyse "eductive" stabilty

4.2.2 "Eductive Stability"

One-dimensional setting. First, consider the one-dimensional system

From the above analysis, it seems natural to make beliefs indexed with growth rates (as underlined in Evans and Guesnerie (2003), beliefs on the proximity of trajectories in the C_0 sense have not enoug grip on the agents'actions.

The hypothetical Common Knowledge assumption, to be taken into account then concerns growth rates.(the C_1 topology).

(Hypothetical) **CK Assumption.** The growth rate of the system is between $\lambda_1 - \epsilon$ and $\lambda_1 + \epsilon$

Such an assumption on growth rates triggers a mental process that, in successful case, progressively reinforces the initial restriction and converges toward the solution. The mental process takes into account the variety of beliefs associated with the initial restriction: common beliefs with point expectations is then a particular case, and it is intuitively plausible that convergence of the general mental process under consideration implies convergence of the special process under examination when studying IE-stability. It is intuitive and in fact quite straightforward in the one-dimensional context that IE-stability is a necessary condition of eductive stability . It follows :

Proposition :(Evans and Guesnerie (2003))

A constant growth rate solution is locally "eductively stable" or "locally strongly rational" then it is determinate in growth rates, locally IE stable, locally immune to susnpots, and attracts all reasonable evolutive learning rules.

Hence "Eductive Stability" is more demanding in general than all the previous equivalent criteria. The fact that it is strictly more is illustrated in the quoted paper, as well as the fact that in the present model it is equally demanding under a behavioural homogeneity condition.

Multi-dimensional setting. In a natural way, the hypothetical Common Knowledge assumption, to be taken into account has to bear on extended growth rates.

(Hypothetical) **CK Assumption.** The extended growth rate of the system B belongs to $V(\overline{B})$.

where $V(\overline{B})$ is a neigbourhood in the space of matrices (that may be defined with respect to some distance evaluated from some matrix norm)

As we said earlier, if $B \in V(\overline{B})$. $\Rightarrow B = \overline{B}$, then the solution is localy "eductively" stable or locally Strongly Rational.

As in the one-dimensional case, one can show

Proposition :(Evans-Guesnerie(2005)

If a *stationnary extended growth rate solution is locally "eductively stable" or "locally strongly rational" then it is determinate, locally IE stable, locally immune to susnpots.*

Again, "Eductive Stability" is more demanding in general than all the previous standard (and as stressed earlier equivalent) criteria.

The reason is that the "eductive" analysis takes into account

1- the stochastic nature of beliefs,

2- the heterogeneity of beliefs, both dimensions which are neglected in the Iterative Expectational stability construct.

In fact, as soon as local "eductive" stability is concerned, the results of Guesnerie-Jara-Moroni (2007), although obtained in a somewha tdifferent context suggest that point-expectations and stochastic expectations do not make so much difference Hence, locally at least, the key differences between Strong rationality and standard expectational stability criteria would come from the heterogeneity of expectations.

4.3 Standard expectational coordination approaches and the "eductive" viewpoint : a tentative conclusion.

First remark. Our attempt at comparing the standard expectational coordination criteria, determinacy, absence of neighbour sunspot equilibria, Iterative Expectational stability, has been limited to a limited class of models. An exhaustive attempt would have to extend the class of models under scrutiny in different directions.

- Introduce uncertainty (intrinsic uncertainty) in the models of previous sections. The analysis should extend, with some technical difficulties, the appropriate objects under scrutiny being then respectively, probability distributions on growth rates (numbers) or extended growth rates (matrices). It is reasonable to conjecture that the above findings would hold somewhat unaffected in the new setting, although the concept of susnpot equilibria should be adapted and extended to take in to account a richer set of extrinsic uncertainty.

- Introduce longer memory lags and/or more forward looking periods. The theory seem applicable although the concept of "extended growth rate" becomes more intricate. (see Gauthier (2004))

Second remark that brings us to the models used in monetary theory.

A number of these models have a structure analogous to the ones under scrutiny before (see next section), although they most often involve intrinsic uncertainty.

This suggest two provisional conclusions that will be put under scrutiny.

1- The standard criterion used in monetary theory for assessing expectational coordination, local determinacy, is less demanding than the "eductive" criterion, because it neglects a dimension of heterogeneity of expectations that is present in the problem.

2- However, the connections between the "evolutive viewpoint" and the "eductive" one is less clearcut than in our prototype model. Differences have three sources :

- the theoretical connection between the two types of learning is less well established in the multidimensional case than in the one-dimensional one. (Proposition 1-3 has no counterpart in Proposition 2)

- In a noisy system, agents do not observe at each step, a state of the system, as defined in our construct, i.e as a probability distribution, but a random realisation drawn from this probability distribution. Learning rules, to be efficient have to react slowly to new information. Intuitively, IE stability and consequently eductive stability will be more demanding local criteria than the criterion of success of, necessarily slow, evolutive learning.

- However, the question of homogeneity of expectations versus heterogeneity and randomness remains.

This is however a conclusion that holds within the framework of truly overlapping generations models. The equations from which the expectational coordination aspects of monetary policy are examined are of the overlapping form but come from infinite horizon models. Their interpretation in an "eductive" analysis is hence different. We will stress this sometimes considerable difference in the next and final Section.

5 Eductive Stability in monetary models.

I will introduce here very simple versions or models that are used for the discussion of monetary policy and of the Central Bank policy. I first introduce a new Keynesian model. I will pursue the discussion in a simpler setting of a cashless economy, in the sense of Woodford (2003).

5.1 Preliminaries on "eductive" stability in a new Keynesian model.

I consider here a new keynesian model, in a linearized reduced form close, but not identical, to that of Woodford (2003), where I forget about noise.

$$\pi_t = bE_t(\pi_{t+1}) + lx_t$$

and

$$x_t = i_t - f(-E_t(\pi_{t+1})) + E_t(x_{t+1})$$

Where

$$i_t = a\pi_{t-1} + cx$$

or

$$i_t = a\pi_t + cx_t$$

Once the interst rate rule is brought into the second equation, the system becomes a one-step forward looking two dimensional model, with or without memory.

The expectational criterion that is used, which leads to stress the Taylor rule $a > 1$, is "determinacy", i.e the fact that there does not exists an infinite sequence meeting the above equations and close to the steady state sequence Previous conditions apply (in the no memory case, the previous condition turn out into a condition on the modulus of the eigenvalues of the relevant matrix, that has to be smallet than one).

The conclusion seems simple.

- The "eductive" viewpoint is in spirit the same as the standard criterion. It is however more demanding since it leads to consider deviations of expectational coordination that relate with the heterogeneity of expectations.

- If, then, one comes back to the underlying model and not to its reduced form, one may wonder whether the agents are"essentially identical in the sense of Evans-Guesnerie (2005), in which case heterogeneity of beliefs may be locally forgotten. I conjecture it is not the case, i.e that a one direction mistake of price setting firms (which are essentially identical) and another direction mistake of the consumershave to be added (this is intuitively why heterogenity matters), but this brings us to the underlying model, and the question is open in the absence of theorem on this issue.

There is however a more basic issue on which I now come.

The equivalence theorems previously stressed are formally proved in an OLG framework. The same holds true for our analysis of the connections between "eductive" stability and standard "expectational" stability. In particular, the "eductive" argument used both for evaluating IE-stability and proving strong rationality take place in "people's minds" but in "OLG people's minds". In a sense, the fact that agents, in the initial model have infinite horizon expectations, so that the "eductive" analysis of expectational coordination must refer to infinite horizon beliefs, has been dropped from the analysis. The main issue is then the following : is it the case that the implicit reduction of beliefs to OLG like beliefs is legitimate from the more basic viewpoint under consideration. In order to clarify this issue, I now focus attention on a model simpler to analyse, a model of a cashless economy, in the spirit of Chapter 2 of Woodford (2003).

5.2 A random walk into "eductive stability' in a cashless economy :

I consider an economy populated by a continuum of identical agents, living for ever. Each agent α receive $\overline{y}$ units of a perishable good at every period. There is money and the good has a money price P_t at each period, The agents have an identical utility function $U = \sum \beta^t u(C_t)$, where $u(C_t)$ will be most often taken as iso-elastic $u(C_t) = [1/(1-\sigma)](C_t)^{(1-\sigma)}$.

First order conditions are $(1+i_t) = (1/\beta)[u'(C_{t+1})/u'(C_t)](P_t/P_{t+1})^{-1} = (1/\beta)(P_{t+1}/P_t)[\frac{C_t}{C_{t+1}}]^{\sigma}$

The Central bank decides on a nominal interest rate according to a Wicksellian rule $i_t^m = \phi(P_t/P_t^*)$,where ϕ is increasing.

As in Woodford, I assume $(P_{t+1}^*) = \overline{\Pi} > \beta$ and $1 + \phi(1) = \overline{\Pi}/\beta$

We note that the path $P_t = P_t^*, C_t(\alpha) = \overline{y}$, define a Rational Expectations, here a perfect foresight, equilibrium.

Is this equilibrium determinate ? It should be noted that, since all agents are similar and face the same conditions in any equilibrium, any equilibrium has to meet $C_t(\alpha) = \overline{y}$. It follows that any other equilibrium P'_t has to meet :

$(1+\phi(P'_t/P^*_t))\beta = (P'_{t+1}/P'_t)$. which can be written $(1+\phi(P'_t/P^*_t))(P'_t/P^*_t)\beta = (P'_{t+1}/P^*_t)$Assume that the other equilibrium is close to the initial one and call $\delta P_t = (P'_t - P^*_t)/P^*_t$. Then, to the first order : $\beta(1+\phi'+\phi)\delta P_t = \delta P_{t+1}$. Hence if $\Pi > 1$, there can be no sequence δP_t meeting this condition and remaining close to P^*_t. The equilibrium is locally determinate.

Note that :

- This may not mean that there are no other perfect foresight equilibria, although the one under scrutiny is the only stationnary one

- If we accept to view the equations as coming from an OLG framework, we would argue that the equilibrium is locally IE-Stable, or even here locally "eductively" stable : the assertion it is CK that a departure in price expectations of δP_{t+1} involves a departure in period t price of δP_t such that $\beta(1+\phi'+\phi)\delta P_t = \delta P_{t+1}$ and if it were CK that $P^*_t + \delta P_t$ remains for ever in a neighbourhood of the equilibrium P^*_t, then a variant of existing argument would involve that the equilibrium * is CK, i.e that it is locally "eductively stable".

However, the first assertion of the just sketched argument, which is a core element of its construction in an OLG framework, makes nos sense here, because the equilibrium condition has to refer to the whole trajectory of beliefs of the agents. To say it in another way, the fact that price expectations to-morrow in period t,is $P^*_t + \delta P_{t+1}$ has no final bite on what the equilibrium price may be to-day in period $t+1$ It has in an OLG framework, wher the period t equilibrium is entirely determined by the beliefs of agents living in period t, on the the characteristics of period $t+1$, the only part of the future inwhich they will live. It is different here : indeed, demand of an agent at period t, as seen from period 0 is :

$C_t(\alpha) = C_1(\alpha)\left[\beta^{t/\sigma}\pi^t_1[(1+i_s)(P_s/P_{s+1})]^{1/\sigma}\right]$. It does depend on the whole agents beliefs over the period and not only on their beliefs over the next period !

The rigth question is then the following : assume that it is the case that hypothetically it is CK that P_s is close to P^*_s then is it the case that the equilibrium is CK. To make the computation easier, I change slightly the Wicksellain rule, replacing $i^m_t = \phi(P_t/P^*_t)$ by $i^m_t = \phi(P_t/P_{t-1})$

The argument has to proceed as follows.

- Express the change of consumption program of an individual as a function of its expectations, for expectations in a neighbourhood of the equilibrium expectations $P^*_t = \overline{\Pi}^{t+1}P^*_0$. Indeed, differentiation of the above formula leads to

$$dC_t/\overline{C} = dC_1/\overline{C} + (\beta/\sigma)\sum_{s=1}^{t} d[1+\phi(P_s/P_{s-1})](P_s/P_{s+1})$$

-To start the "edcutive satbility argument, assume that all agents believe that inflation in the future will be $\overline{\Pi} + \ \epsilon$, and let us check waht will happen in period 1, given these beliefs. (I denote $\phi' = v$)

We have

$$dC_t/\overline{C} = dC_1/\overline{C} + (\beta/\sigma)[v(P_1/P_0) + \sum_{s=2}^{t} \overline{1/\Pi}(v - 1/\beta)\epsilon$$

$$= dC_1/\overline{C} + (\beta/\sigma)[v(P_1/P_0) + (t-1)(\overline{1/\Pi}(v - 1/\beta)\epsilon$$

An enveloppe argument (the fact that the utility of agents is unaffected to the first order by this change of beliefs) implies

$$dC_1 + [v/\sigma(P_1/P_0^*)] \sum_{t=1}^{+\infty}(\beta^{t+1}) + [(\overline{1/\Pi\sigma)}(v - 1/\beta)\epsilon] \sum_{t=1}^{+\infty}(t-1)(\beta^{t+1}) = 0$$

i.e

$$dC_1 + [v/\sigma(P_1/P_0^*)](\beta/(1-\beta) + \beta^3/(1-\beta)^2[(1/\overline{\Pi}\overline{\sigma}(v - 1/\beta)\epsilon] = 0$$

Now equilibrium on the first period market, given these beliefs involve :

$$[(P_1/P_0^*)/\overline{\Pi}] + \beta^2/(1-\beta)[((1 - 1/\beta v)\epsilon] = 0$$

The formula suggest that first period realised inflation goes the other way, but much outside the conjectured band of increased inflation. This suggests that the infinite horizon equilibrium is not "eductively" stable, for every positive v at least from a somewhat mechanical, too mechanical, view of the mental process (the best v seems to be $1/\beta$) .

Note also, that contrarily to what happens in a standard RBC model, (Evans, Guesnerie, Mc Gough, (2007), work in progress) the intertemporal elasticity of substitution does not play a role

5.3 Conclusion.

The conclusion is necessarily provisional, since the outsider's random walk in monetary models although from a well delineated basis, has to be confronted with the criticism and enriched by an intuition somewhat missing in the present state of my understanding of the specialized issues that have been touched.

It seems however that this otusider's walk may raise interesting questions for insiders and then opens new fronts of thinking. It is at least a reasonable hope at this stage.

6 Bibliography

1. B.D. Bernheim, Rationalizable strategic behavior, Econometrica 52 (1984), 1007-28.

2. P.A. Chiappori, R. Guesnerie, Sunspot equilibria in sequential markets models, in:W. Hildenbrand and H. Sonnenschein (Eds.), Handbook in Mathematical Economics, Vol. 4, North Holland, Amsterdam, 1991, pp. 1683-1762.

3. Chamley, Christophe P. 2004. *Rational Herds: Economic Models of Social Learning*. Cambridge University Press.

4. Chiappori, P.A., Geoffard, P.Y. and R. Guesnerie, 1992, Sunspot fluctuations around a steady state: The case of multidimensional one-step forward looking economic models, Econometrica 60, 1097-1126.

5. Dávila, J., 1997, Sunspot fluctuations in dynamics with predetermined variables, Economic Theory 10, 483-495.

6. Desgranges, Gabriel. 2000, August. "CK-Equilibria and Informational Efficiency in a Competitive Economy". Econometric society world congress 2000 contributed papers 1296, Econometric Society. available at http://ideas.repec.org/p/ecm/wc2000/12

7. Desgranges, Gabriel, and Maik Heinemann. 2005, March. "Strongly rational expectations equilibria with endogenous acquisition of information". Working paper series in economics 9, University of L¨uneburg, Institute of Economics. available at http://ideas.repec.org/p/lue/wpaper/9.html.

8. Desgranges, G. and S. Gauthier, 2003, On uniqueness of the bubble-free solution in linear rational expectations models, Macroeconomic Dynamics 7, 171-191.

9. DeCanio, Stephen J. 1979. "Rational Expectations and Learning from Experience". *The Quarterly Journal of Economics* 93 (1): 47–57 (Feb.).

10. Evans, G Expectational stability and the multiple equilibrium problem in linear rational expectations models, Quarterly Journal of Economics 100 (1985), 1217-33.

11. Evans, G, Expectations in economics: adaptive versus eductive learning, Revue Économique 52 (2001), 573-582.

12. Evans, G and R. Guesnerie, Rationalizability, strong rationality and expectational stability, Games and Economic Behaviour 5 (1993), 632-646.

13. Evans, G. and R. Guesnerie, 2003a, Coordination on Saddle Path Solutions: The Eductive Viewpoint - Linear Univariate Models, Macroeconomic Dynamics 7, 42-62.

14. Evans, G. and R. Guesnerie, 2005, Coordination on saddle-path solutions: The eductive viewpoint. Part 2 - linear multivariate models, Journal of Economic Theory, 124, 202-229

15. Evans, G, R. Guesnerie, and B. Mc Gough 2007 "Eductive stability in a RBC setting", work inprogress

16. Evans, G. and S. Honkapohja, 2001, Learning and Expectations in Macroeconomics, Princeton University Press, Princeton, New Jersey, 2001.

17. Gauthier, S., 2002, Determinacy and stability under learning of rational expectations equilibria, Journal of Economic Theory 102, 354-374.

18. Gauthier, S., 2003, On the dynamic equivalence principle, Macroeconomic Dynamics 7, 63-88.

19. Gauthier, S., 2004, On determinacy and perfect foresight, Journal of Mathematical Economics

20. Gauthier S. and R. Guesnerie, 2005, Chapter 11 in Guesnerie (2005).

21. Grandmont, J.M. and G. Laroque, 1991, Economic dynamics with learning: Some instability examples, in: W. Barnett, B. Cornet, C. d'Aspremont, J. Gabszewicz, and A. Mas-Colell, Eds., Equilibrium Theory and Applications (Cambridge, U.K.: Cambridge University Press) 247-273

22. Guesnerie and Jean-Charles Rochet. 1993 "(De)stabilizing speculation on futures markets : An alternative view point". *European Economic Review* 37 (5): 1043–1063

23. Guesnerie, R. and M. Woodford, 1991, Stability of cycles with adaptive learning rules, in: W.A. Barnett et al., Eds., Equilibrium Theory and Applications: Proceedings of the Sixth International Symposium in Economic Theory and Econometrics, Cambridge U.K., Cambridge University Press.

24. Guesnerie R. and M. Woodford, 1992, Endogenous fluctuations, in: J.J. Laffont, Ed., Advances in Economic Theory, 289-412, Cambridge University Press.

25. Guesnerie, R, An exploration of the eductive justifications of the rational expectations hypothesis, American Economic Review 82 (1992), 1254-1278.

26. Guesnerie R. (2001) “"Short run expectational coordination: Fixed versus flexible wages." Quarterly Journal of Economics, p. 1115,1147.

27. Guesnerie, R. Anchoring economic predictions in common knowledge, Econometrica 70 (2002), 439-480.

28. Guesnerie R. (2005) "Assessing Rational Expectations 2 : Eductive Stability in economics", MIT Press, , 455p.

29. Lucas, Robert E., Jr. 1978. “Asset Prices in an Exchange Economy”. *Econometrica* 46 (6): 1429–1445 (November).

30. B.T. McCallum, On nonuniqueness in linear rational expectations models: an attempt at perspective, Journal of Monetary Economics 11 (1983), 139-168.

31. Milgrom, Paul, and John Roberts. 1990. “Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities”. Econometrica 58 (6): 1255–1277 (November).

32. H. Moulin, Dominance solvable voting schemes, Econometrica 47 (1979), 1337-1351.

33. Morris, Stephen, and Hyun Song Shin. 1998. “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks”. *American Economic Review* 88 (3): 587–97 (June).

34. D. Pearce, Rationalizable strategic behavior and the problem of perfection, Econometrica 52 (1984), 1029-50.

35. Sargent T. and Wallace N; (1975) "Rational Expectations, the optimal monetary instrument and the optimal money supply", Journal of Political Economy, 83, 251-254.

36. P. Reichlin, Equilibrium cycles in an overlapping generations economy with production, Journal of Economic Theory 40 (1986), 89-102.

37. Tan, Tommy Chin-Chiu, and Sérgio Ribeiro da Costa Werlang. 1988. “The Bayesian Foundations of Solution Concepts of Games”. *Journal of Economic Theory* 45 (2): 370–391 (August).

38. Taylor J. (ed) 1999, Monetary Policy Rules, University of Chicago Press.

39. Woodford M. (2003) “Interest and Prices : Foundations of theory of Monetary Policy”, Princeton University Press.